Gravitation

constants, a wall, and some waves

Jean-Philippe UZAN

Testing General Relativity & EEP

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

Physical metric

$$S_{matter}(\psi, g_{\mu\nu})$$

gravitational metric

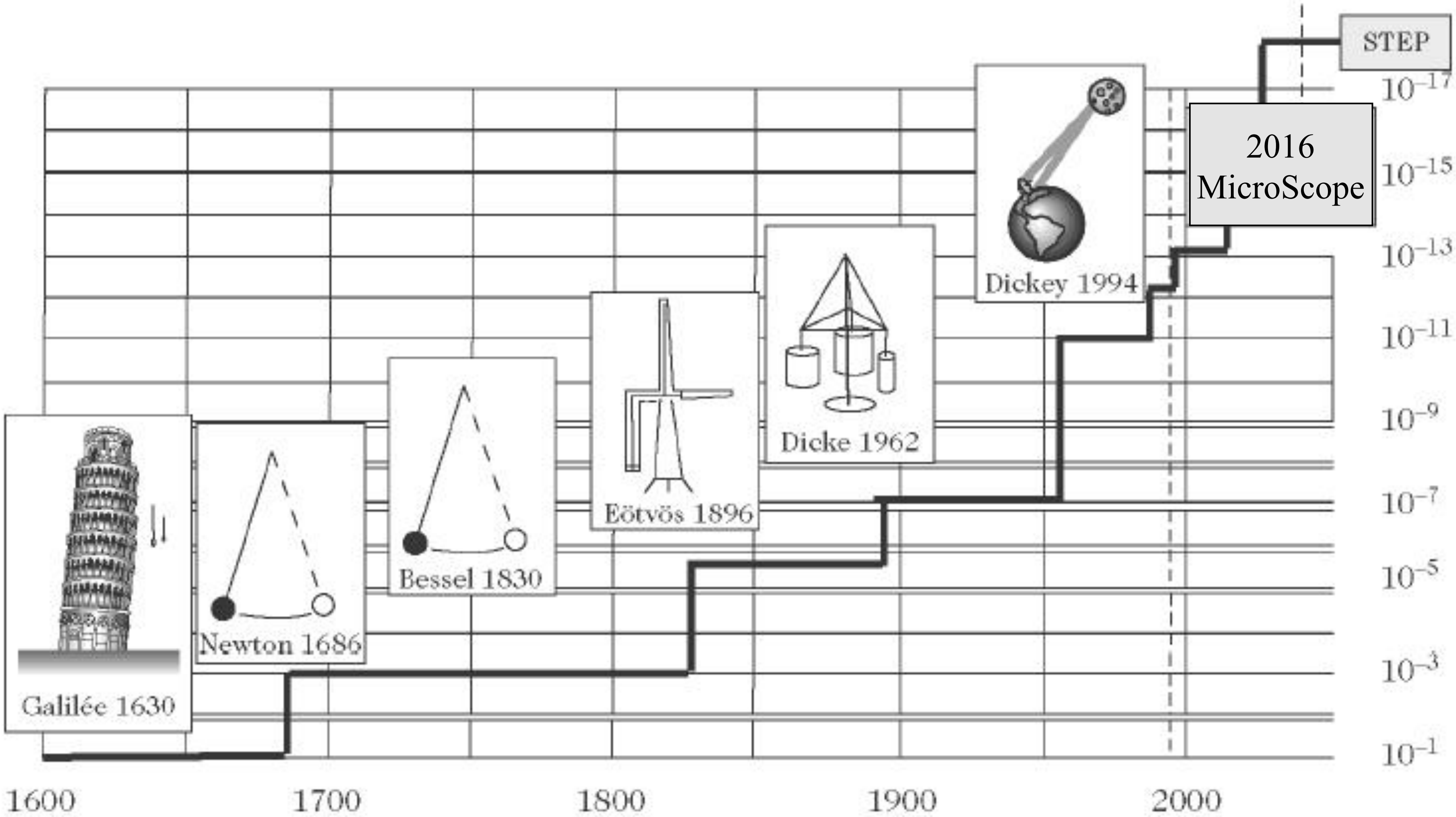
Dynamics

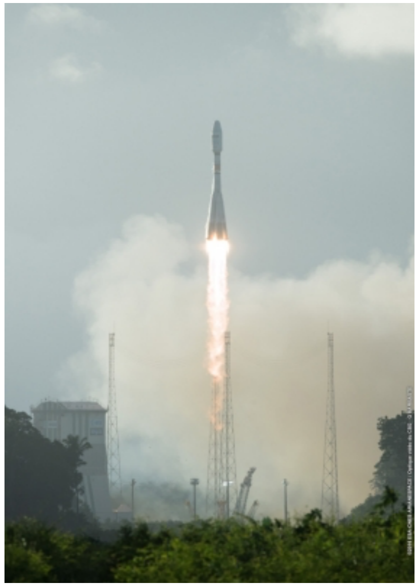
$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-\,g_*}\,R_*\,d^4x$$

Relativity

$$\boxed{g_{\mu\nu} = g^*_{\mu\nu}}$$

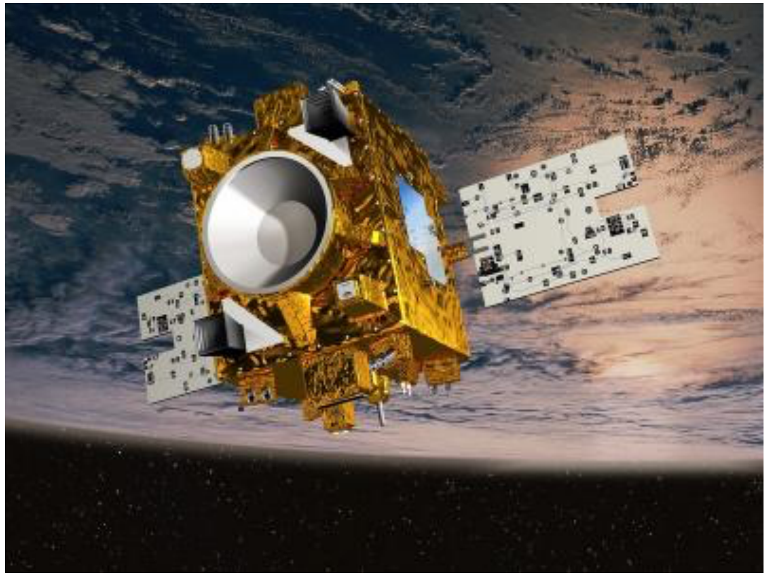
Tests on the universality of free fall

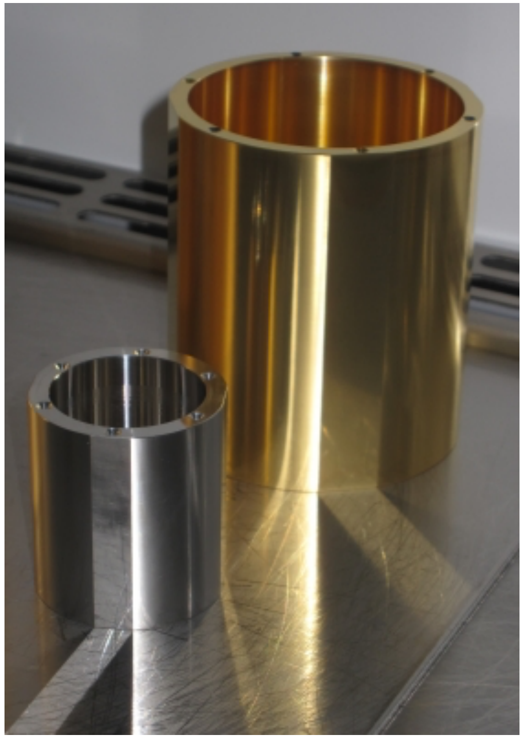


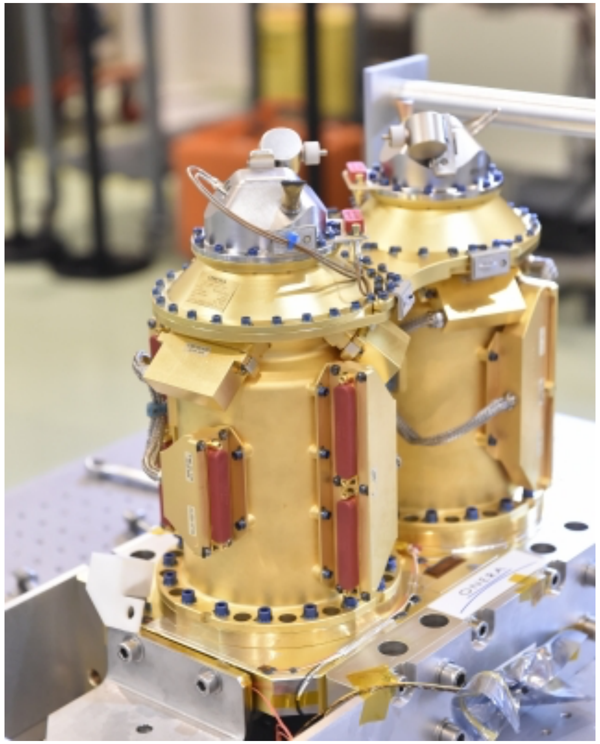


MICROSCOPE
(CNES)

Lundi 25 avril 2016 à
18h02 (heure locale),
Soyouz @ Kourou







Bergé, Pernot-Borras, JPU et al, 2017

Scalar-tensor theories

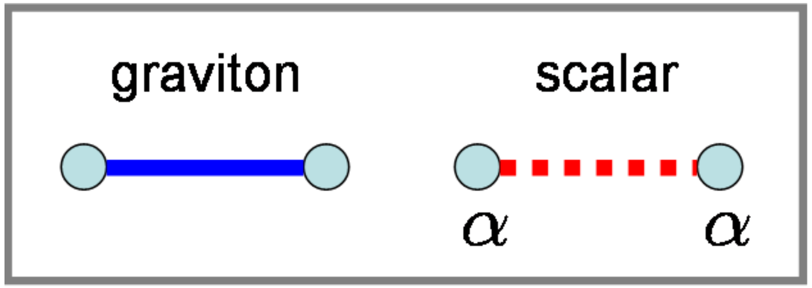
Most general theories of gravity that include a scalar field beside the metric

- Mathematically **consistent**
- Motivated by **superstring**
  - **dilaton** in the graviton supermultiplet,
  - **modulii** after dimensional reduction
- Consistent field theory to satisfy WEP
- Useful extension of GR (simple but general enough)

spin 2

spin 0

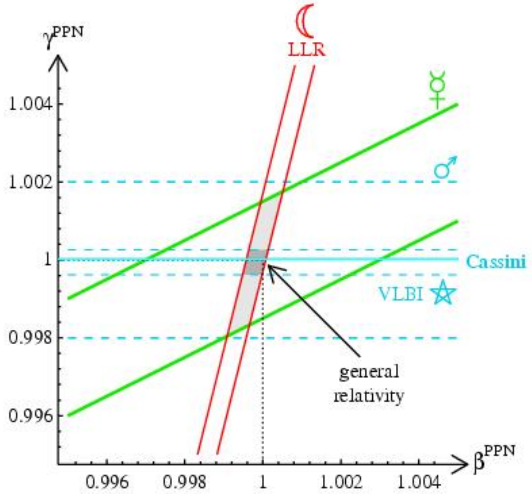
$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\}$$
$$+ S_m\{\mathrm{matter}, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}\}$$

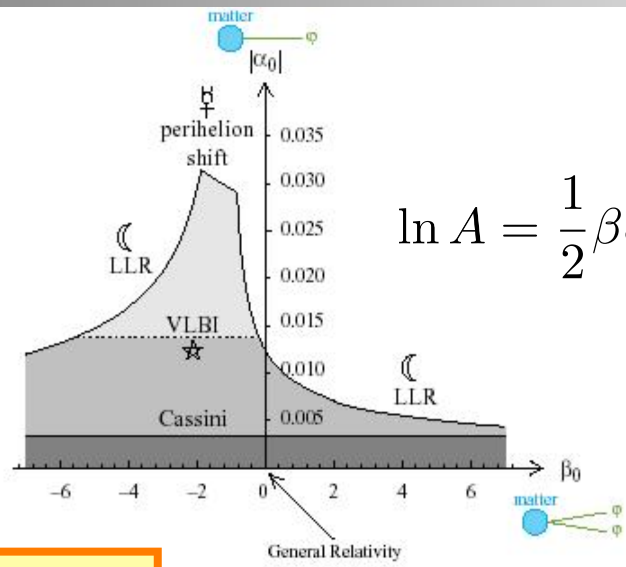


$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$
$$\beta = \mathrm{d}\alpha/\mathrm{d}\phi$$

Deviation from GR

Courtesy of Esposito-Farèse



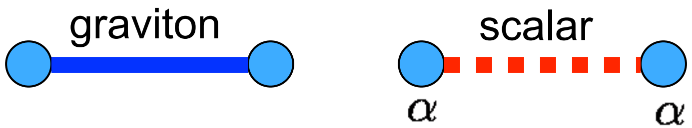


$$\ln A = \frac{1}{2}\beta\phi^2$$

$$\alpha_0^2 < 10^{-5}, \qquad -4.5 < \beta_0$$

Variation of G

$$G_{\rm cav} = G(1+\alpha^2)$$



$$\left.\frac{\dot G}{G}\right|_0 \equiv \sigma_0 H_0$$

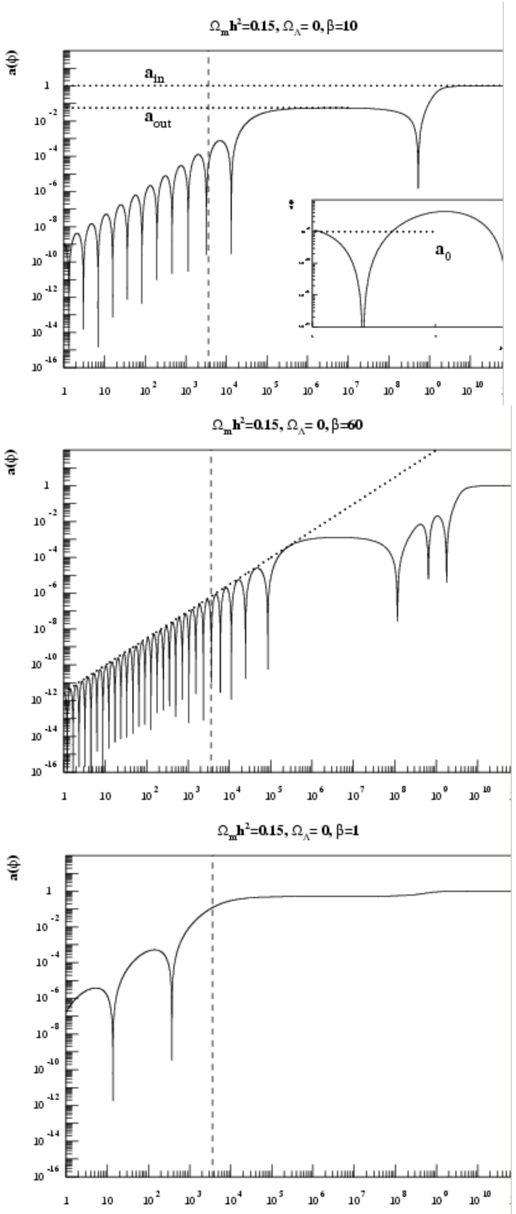
$$\frac{\dot G}{G} < 10^{-12}\,\mathrm{yr}^{-1}$$

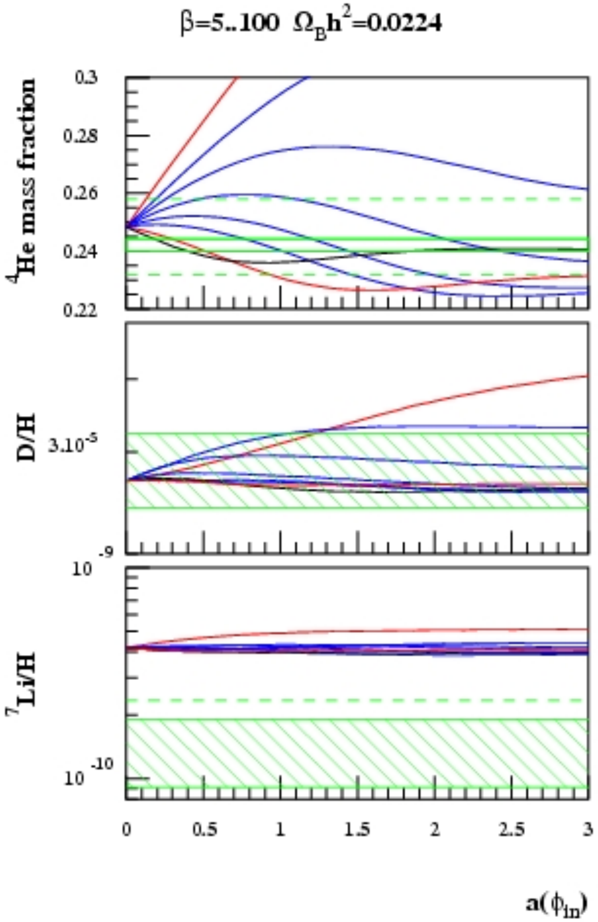
$$\sigma_0 < 10^{-2}$$

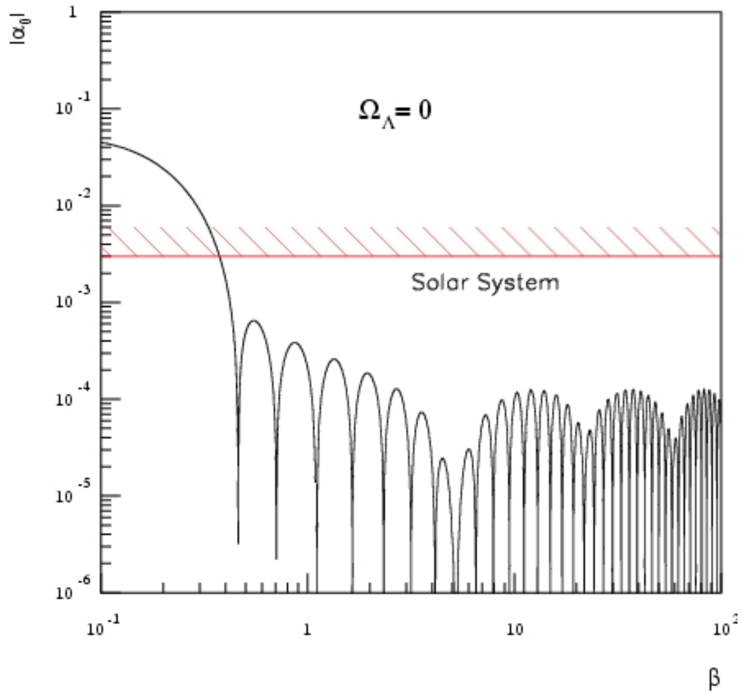
Constraints valid for a (almost) massless field.

Scalar-tensor theories

Full dynamics ⟶ Abundances ⟶ constraints







Coc Olive, JPU, Vangioni. 2006

BBN – scalar-tensor and beyond

- Contraints on scalar-tensor theories from BBN

  Coc Olive, JPU, Vangioni. 2006

- Quantum corrections to the evolution of the scalar mode

  Cembrano Olive, JPU, Peloso. 2009

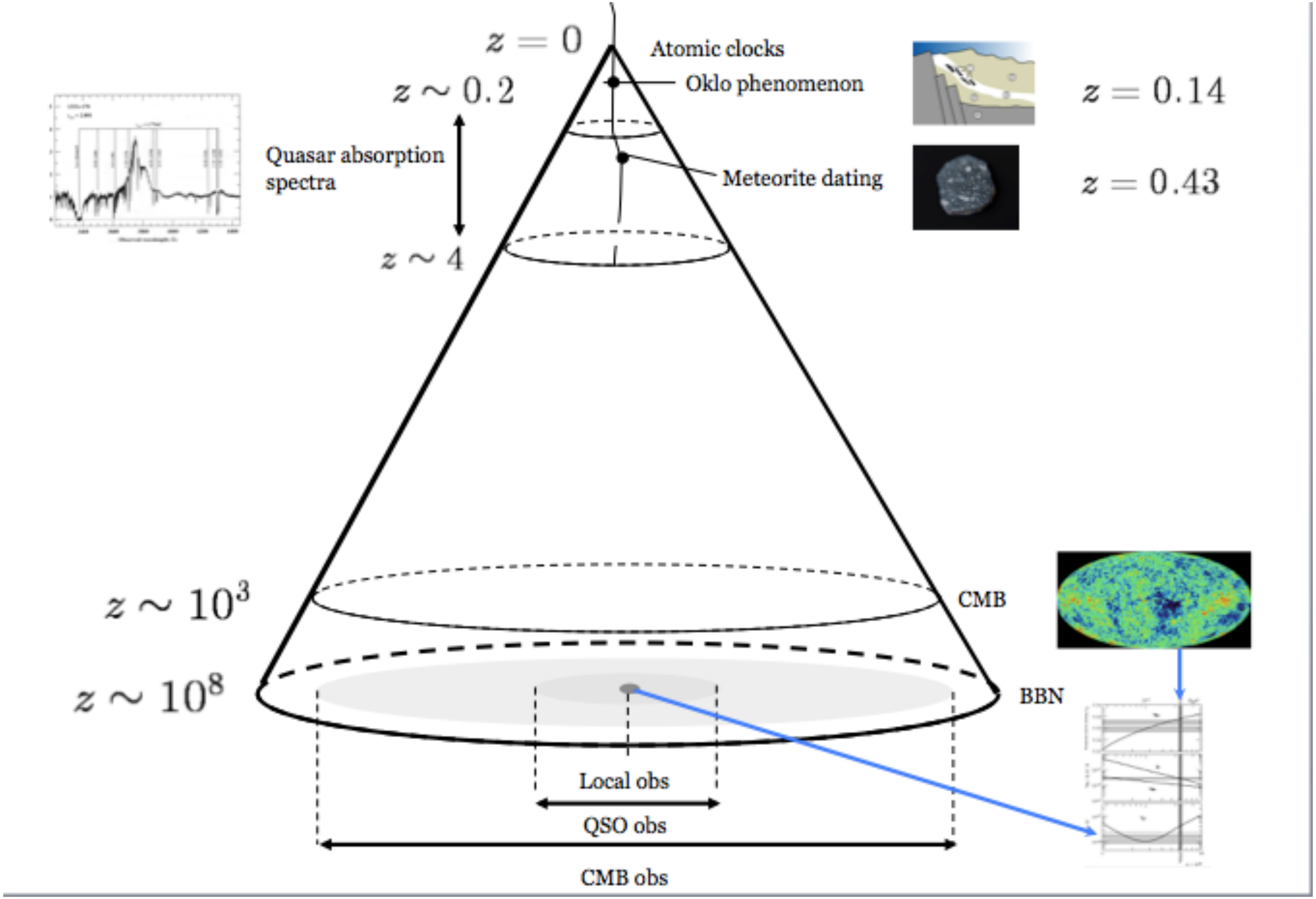
- Extension to a differential coupling between baryonic / DM sector

  Coc, Olive, JPU, Vangioni. 2009

- Non universal couplings

  Coc Nunes, Olive, JPU, Vangioni. 2007

Physical systems



Constancy of fundamental constants

A given physical system gives us an observable quantity

$$O(G_k, X)$$

External parameters: *temperature,....:*

Primary physical parameters

Step 1:

From a physical model of our system we can deduce the sensitivities to the primary physical parameters

$$\boxed{\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k}}$$

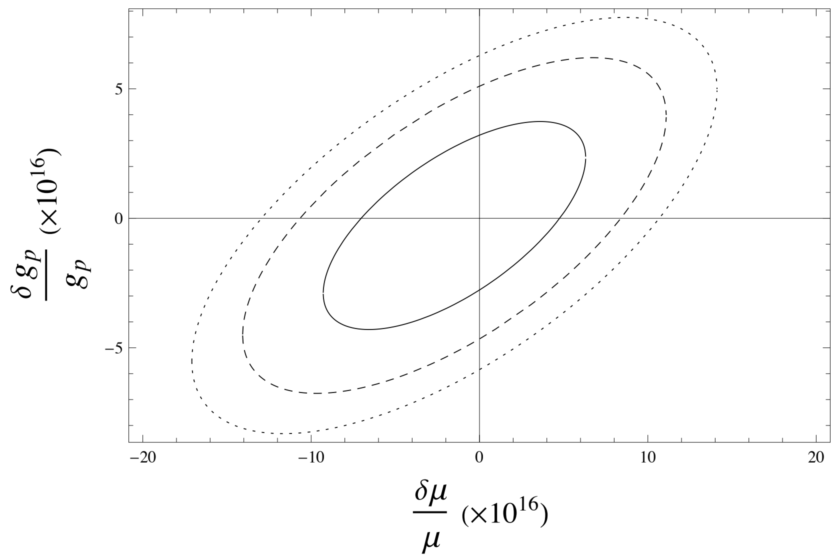
Step 2:

The primary physical parameters are usually not fundamental constants.

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i$$

Local vs cosmological

Clock 1	Clock 2 $\frac{\mathrm{d}}{\mathrm{d}t}\ln\left(\frac{\nu_{\mathrm{clock}_1}}{\nu_{\mathrm{clock}_2}}\right)$	Constraint (yr^{-1})	Constants dependence	Reference
^{87}Rb	^{133}Cs	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{\mathrm{Cs}}}{g_{\mathrm{Rb}}}\alpha_{\mathrm{EM}}^{0.49}$	[13]
^{87}Rb	^{133}Cs	$(-0.5 \pm 5.3) \times 10^{-16}$		[14]
^{1}H	^{133}Cs	$(-32 \pm 63) \times 10^{-16}$	$g_{\mathrm{Cs}}\mu\alpha_{\mathrm{EM}}^{2.83}$	[15]
^{199}Hg^{+}	^{133}Cs	$(0.2 \pm 7) \times 10^{-15}$	$g_{\mathrm{Cs}}\mu\alpha_{\mathrm{EM}}^{6.05}$	[16]
^{199}Hg^{+}	^{133}Cs	$(3.7 \pm 3.9) \times 10^{-16}$		[17]
^{171}Yb^{+}	^{133}Cs	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{\mathrm{Cs}}\mu\alpha_{\mathrm{EM}}^{1.93}$	[18]
^{171}Yb^{+}	^{133}Cs	$(-0.78 \pm 1.40) \times 10^{-15}$		[22]
^{87}Sr	^{133}Cs	$(-1.0 \pm 1.8) \times 10^{-15}$	$g_{\mathrm{Cs}}\mu\alpha_{\mathrm{EM}}^{2.77}$	[19]
^{87}Dy	^{87}Dy			[20]
^{27}Al^{+}	^{199}Hg^{+}	$(-5.3 \pm 7.9) \times 10^{-17}$	$\alpha_{\mathrm{EM}}^{-3.208}$	[21]

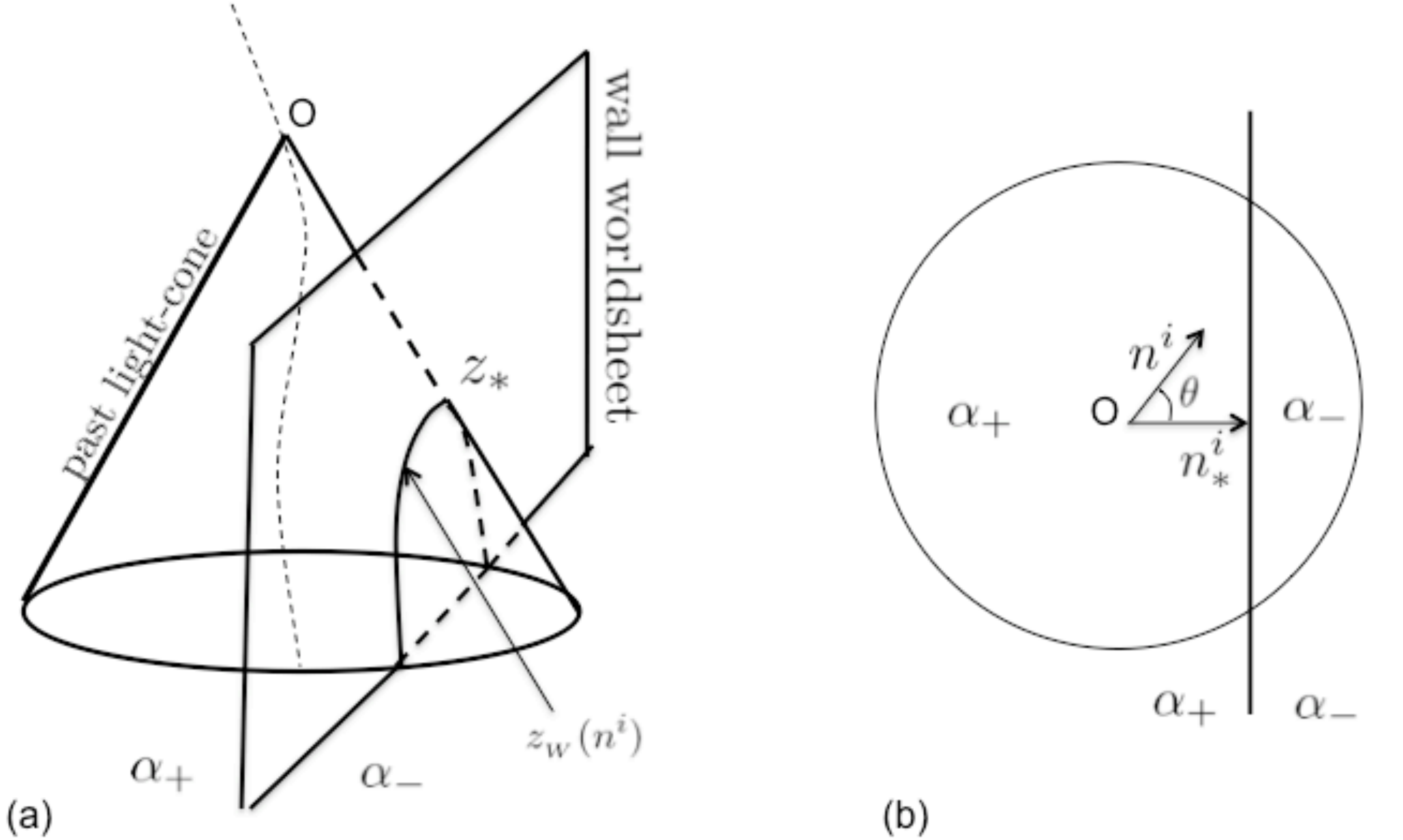


[Luo, Olive, JPU, 2011]

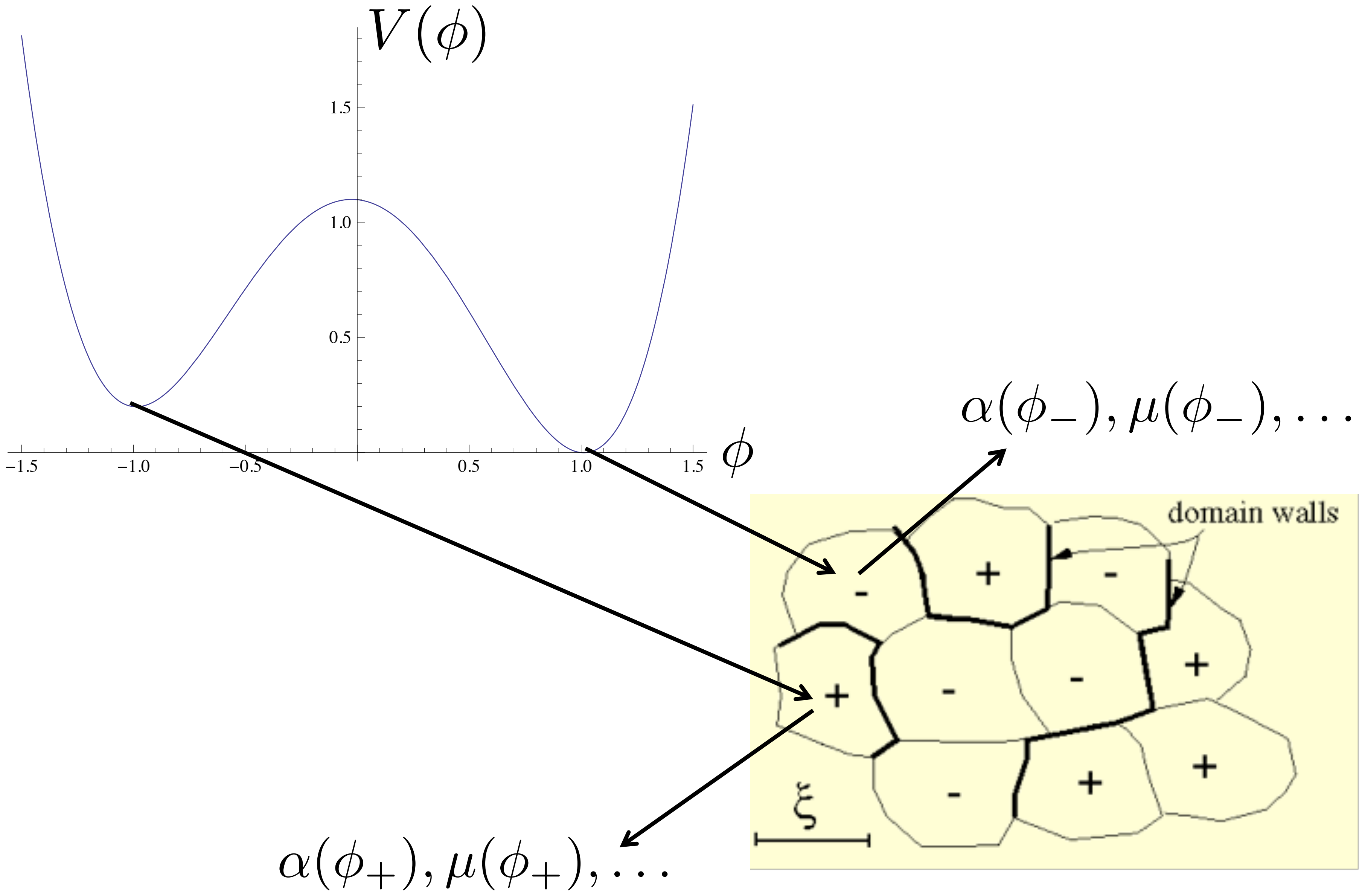
Local vs global

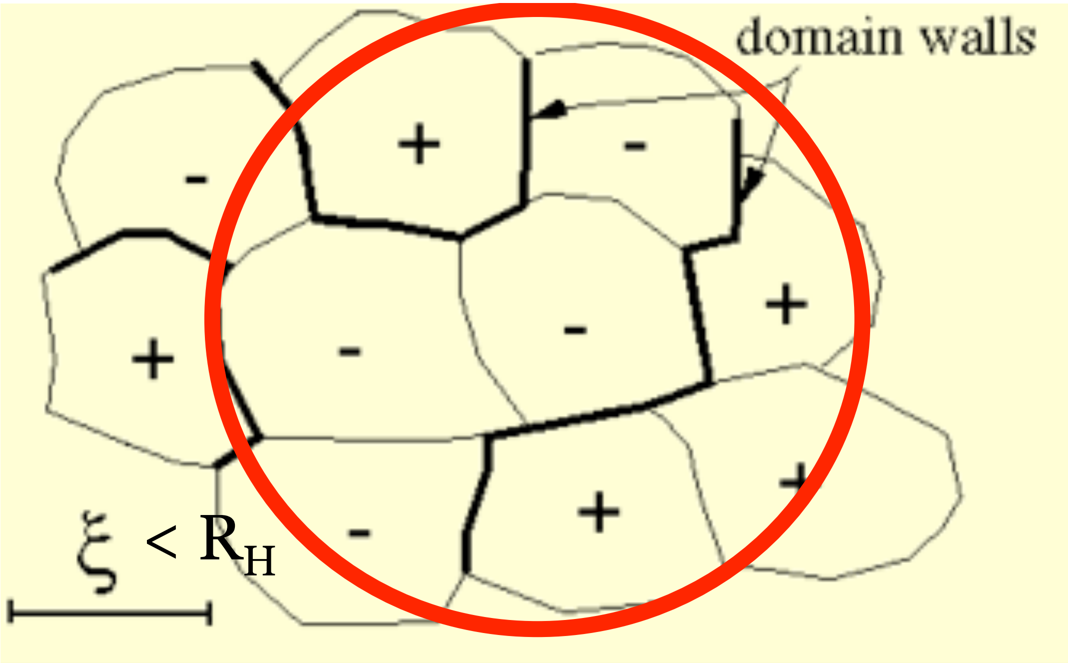
[Olive, Peloso, JPU, 2010]

Idea: Spatial discontinuity in the fundamental constant due to a domain wall crossing our Hubble volume.

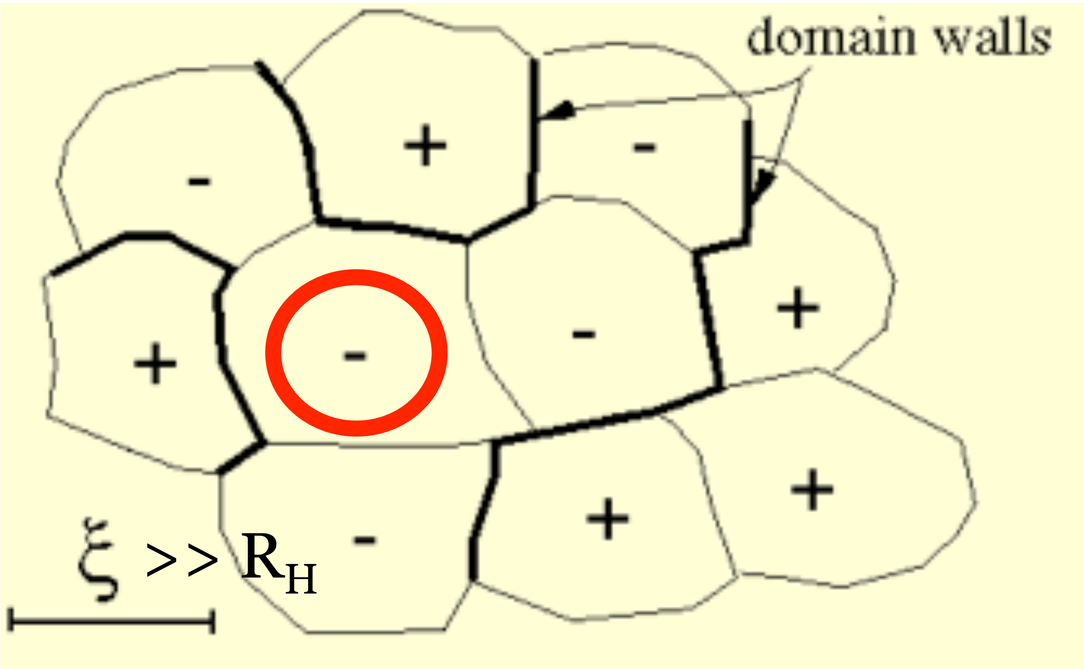


Spatial distribution of the constants





Constants vary on sub-Hubble scales.
 - may be detected
 - microphysics in principle acessible



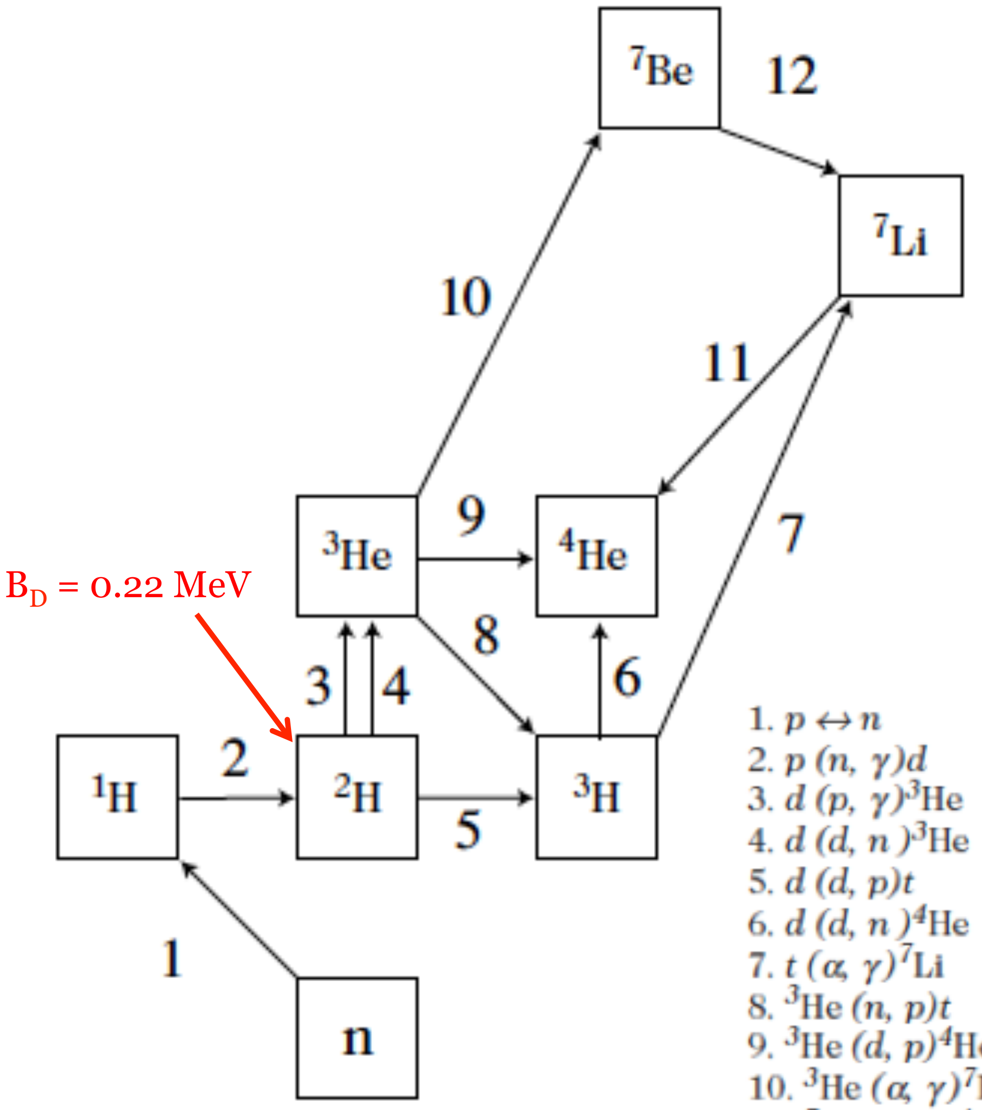
Constants vary on super-Hubble scales.
 - landscape ?
 - exact model of a theory which dynamically gives a distribution of fondamental constants
 - no variation on the size of the observable universe

[JPU, 2011]

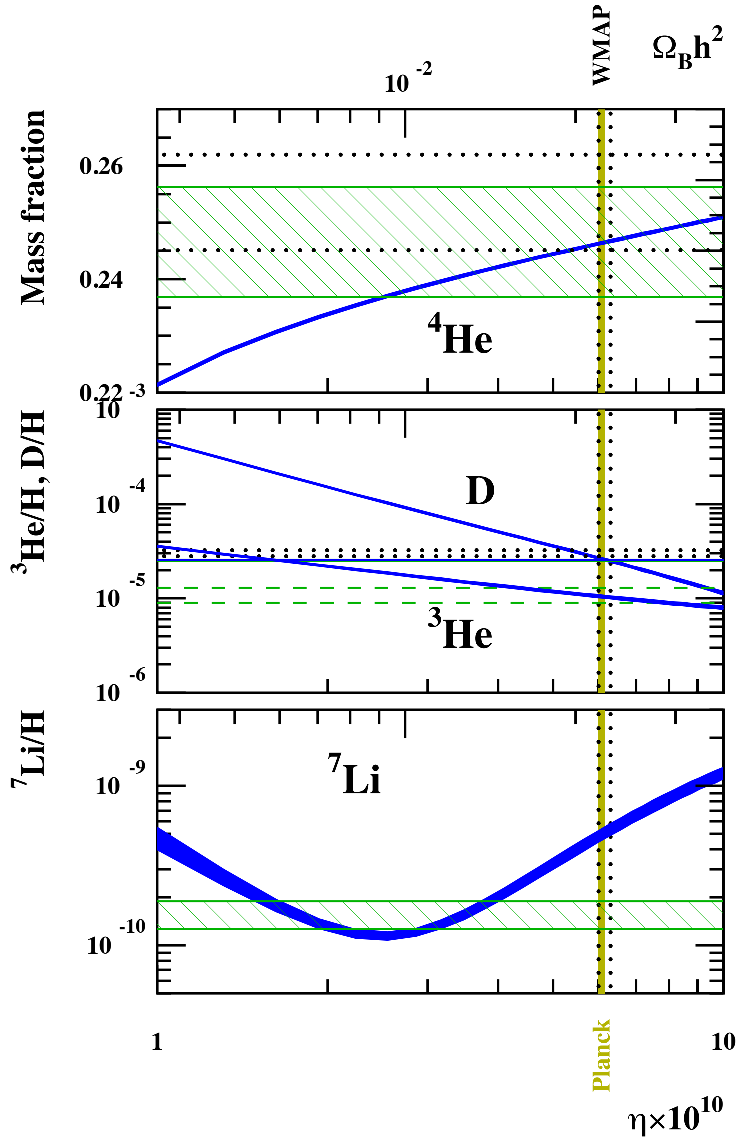
Nuclear physics in astrophysical context

- Big bang nucleosynthesis
- Stellar nucleosynthesis

BBN



1. $p \leftrightarrow n$
2. $p\,(n,\gamma)d$
3. $d\,(p,\gamma)^3$He
4. $d\,(d,n)^3$He
5. $d\,(d,p)t$
6. $d\,(d,n)^4$He
7. $t\,(\alpha,\gamma)^7$Li
8. ^{3}He $(n,p)t$
9. ^{3}He $(d,p)^4$He
10. ^{3}He $(\alpha,\gamma)^7$Be
11. ^{7}Li $(p,\alpha)^4$He
12. ^{7}Be $(n,p)^7$Li



Stellar evolution – 3α

Triple α coincidence (Hoyle)

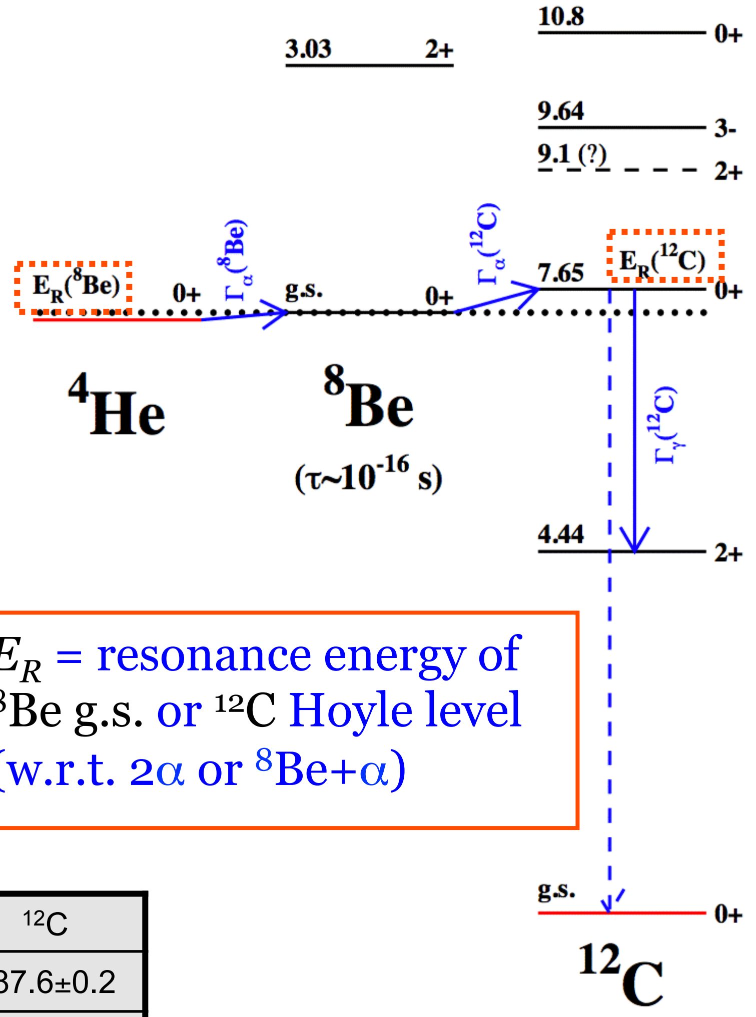
1. Equillibrium between ^{4}He and the short lived (~10^{-16} s) ^{8}Be : αα↔^{8}Be
2. Resonant capture to the (l=0, J^π=0$^+$) Hoyle state: ^{8}Be+α→^{12}C*(→^{12}C+γ)

Simple formula used in previous studies

1. Saha equation (thermal equilibrium)
2. Sharp resonance analytic expression:

$$N_A^2\langle\sigma v\rangle^{\alpha\alpha\alpha} = 3^{3/2}6N_A^2\left(\frac{2\pi}{M_\alpha \mathrm{k_B}T}\right)^3\hbar^5\gamma\exp\left(\frac{-Q_{\alpha\alpha\alpha}}{\mathrm{k_B}T}\right)$$

with $Q_{\alpha\alpha\alpha}= E_R(^8\mathrm{Be}) + E_R(^{12}\mathrm{C})$ and $\gamma\approx\Gamma_\gamma$



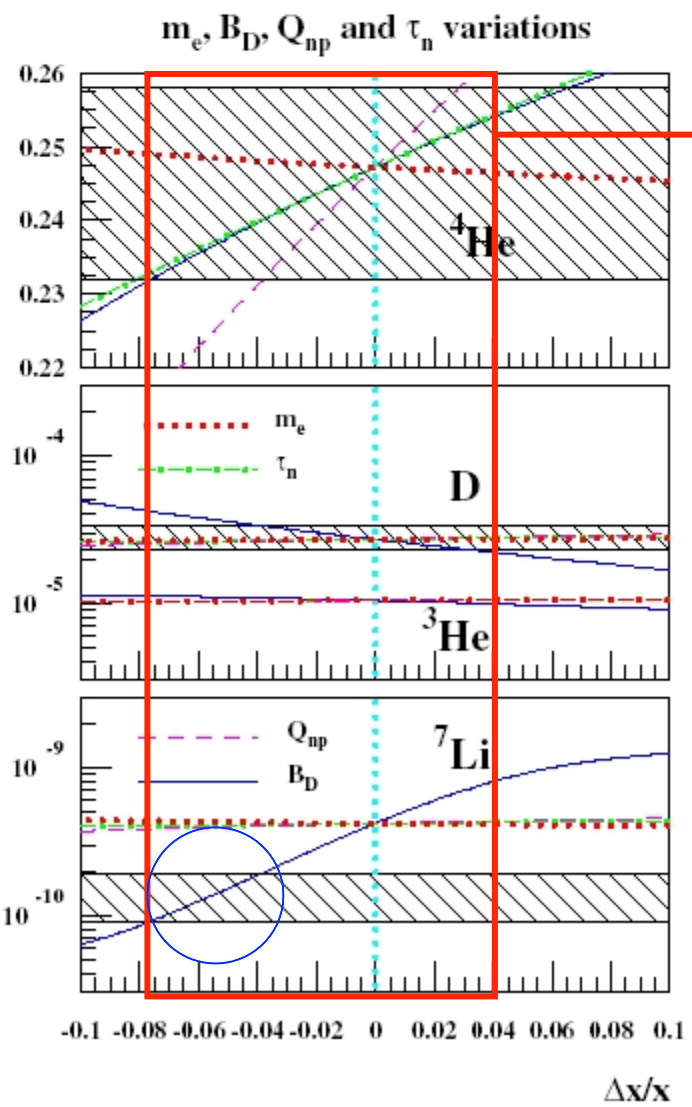
E_R = resonance energy of ^{8}Be g.s. or ^{12}C Hoyle level (w.r.t. 2α or ^{8}Be+α)

Nucleus	^{8}Be	^{12}C
E_R (keV)	91.84±0.04	287.6±0.2
Γ_α (eV)	5.57±0.25	8.3±1.0
Γ_γ (meV)	-	3.7±0.5

[Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni,2009]

BBN

Independent variations of the BBN parameters



$$-7.5\times10^{-2}<\frac{\Delta B_D}{B_D}<6.5\times10^{-2}$$

$$-8.2\times10^{-2}<\frac{\Delta\tau_n}{\tau_n}<6\times10^{-2}$$

$$-4\times10^{-2}<\frac{\Delta Q}{Q}<2.7\times10^{-2}$$

Abundances are very sensitive to B_D.
Equilibrium abundance of D and the reaction rate p(n,γ)D depend exponentially on B_D.

These parameters are not independent.

Difficulty: QCD and its role in low energy nuclear reactions.

Coc,Nunes,Olive,JPU,Vangioni 2007

Constraints

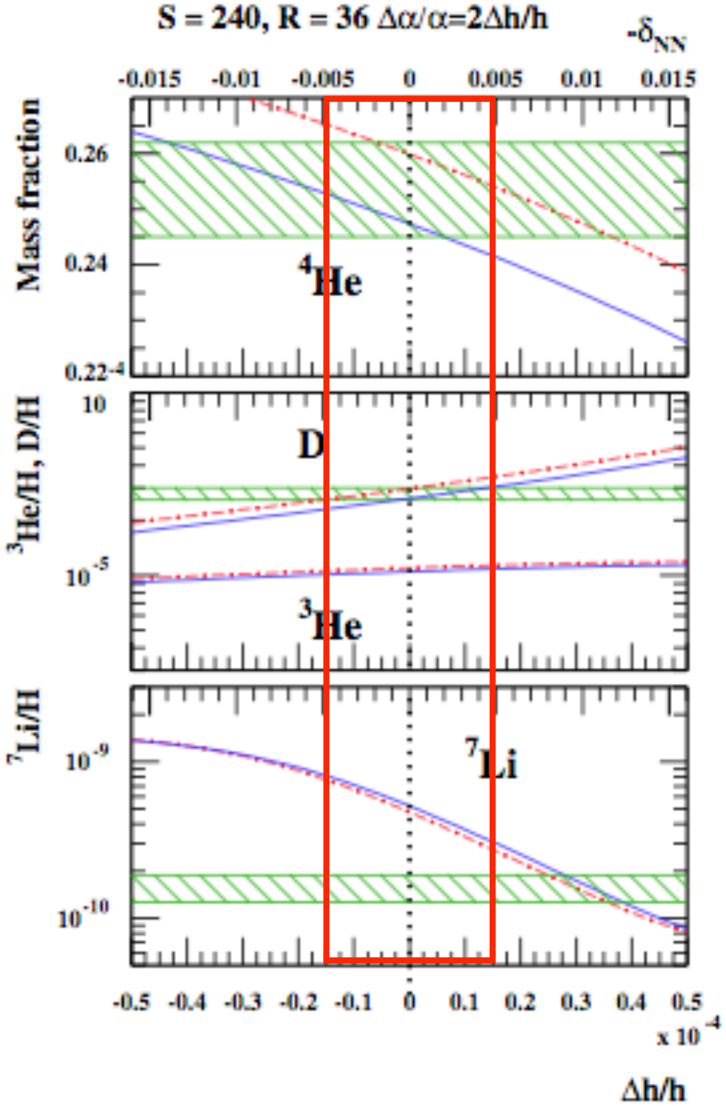
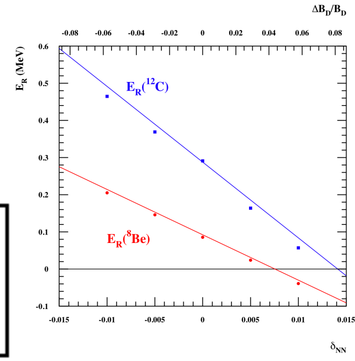


FIG. 12 (color online). Update Fig. 4 of Ref. [22] assuming $S = 240$ and $R = 36$ (solid blue line), using new rates for $^3He(\alpha,\gamma)^7Li$ [73] and $^1H(n,\gamma)D$ [74] and the Ω_b value from WMAP7 [4]. The top axis is $-\delta_{NN}$ from Eq. (5.8) (mind the sign) and the dashed red line assumes $N_\nu = 4$.

Coc, Descouvemont,Olive, JPU,Vangioni 2012

Stellar evolution – 3α



Fundamental constants

$\alpha_i \supset \{\alpha, h_a, v, \ldots\}$

Step I

Nuclear parameters

$Q_i \supset \{B_D, E_R(^8\text{Be}), E_R(^{12}\text{C}) \ldots\}$

Effective N-N interaction
Minnesota model
Parameters: $(\delta_\alpha, \delta_{NN})$

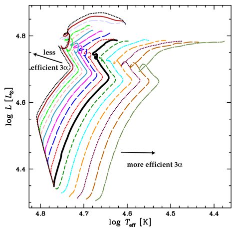
Step II
Integration over energy

- Nuclear abundances,
(^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ...)
- Lifetime of stars, HR diagram

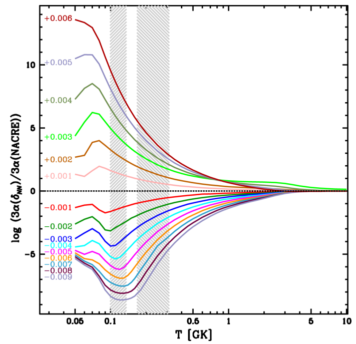
Step III

Reaction rates

$\sigma_i(T, Q_j), \quad \lambda_i(T, Q_j)$



Mass of the star
Initial metallicity
Rotation?
Mass loss?

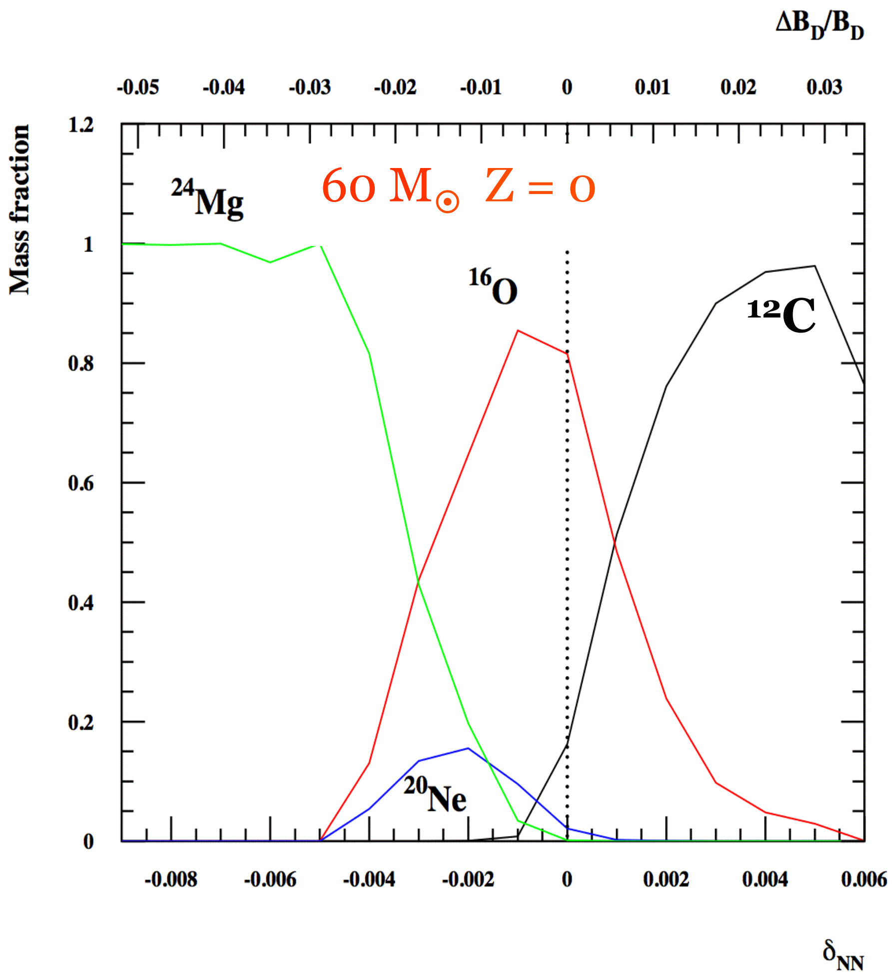


Constraints on the variation of constants

Stellar evolution – 3α

Stellar evolution of massive Pop. III stars

*We choose **typical** masses of 15 and 60 $M_\odot$ stars/ Z=0 ⇒Very specific stellar evolution*



- **The standard region:** Both ^{12}C and ^{16}O are produced.
- **The ^{16}O region:** The 3α is slower than $^{12}C(\alpha,\gamma)^{16}O$ resulting in a higher T_C and a conversion of most ^{12}C into ^{16}O
- **The ^{24}Mg region:** With an even weaker 3α, a higher T_C is achieved and $^{12}C(\alpha,\gamma)^{16}O(\alpha,\gamma)^{20}Ne(\alpha,\gamma)^{24}Mg$ transforms ^{12}C into ^{24}Mg
- **The ^{12}C region:** The 3α is faster than $^{12}C(\alpha,\gamma)^{16}O$ and ^{12}C is not transformed into ^{16}O

Constraint **$^{12}C/^{16}O \sim 1$**

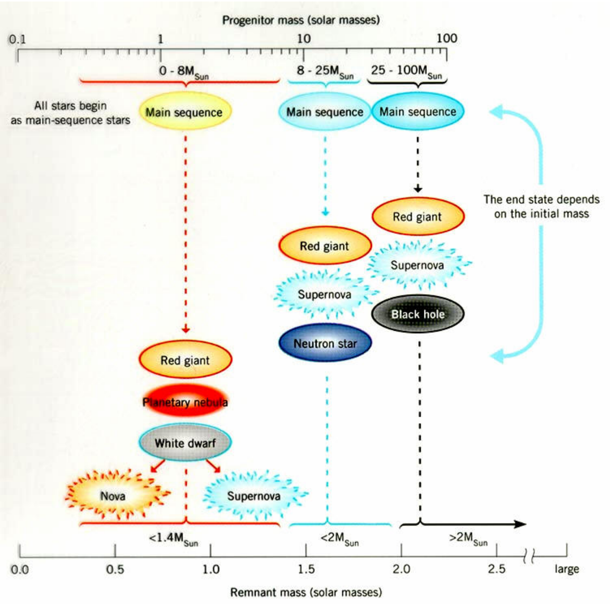
$-0.0005 < \delta_{NN} < 0.001$

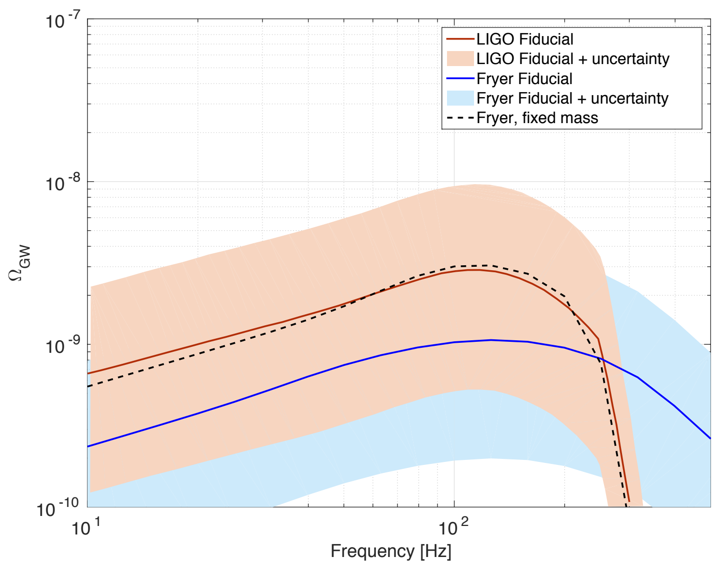
or

$-0.003 < \Delta B_D/B_D < 0.009$

[Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni,2009]

From pop. III to BH & GW





Pop. III stars ends in either NS or BH

- evolution depends on (M,Z)

- Study of the BH merger rate (and GW background) & metallicity.

- dependence in
  - initial mass function
  - star formation rates
  - BH mass prediction

- predict also Z(t), reionisation

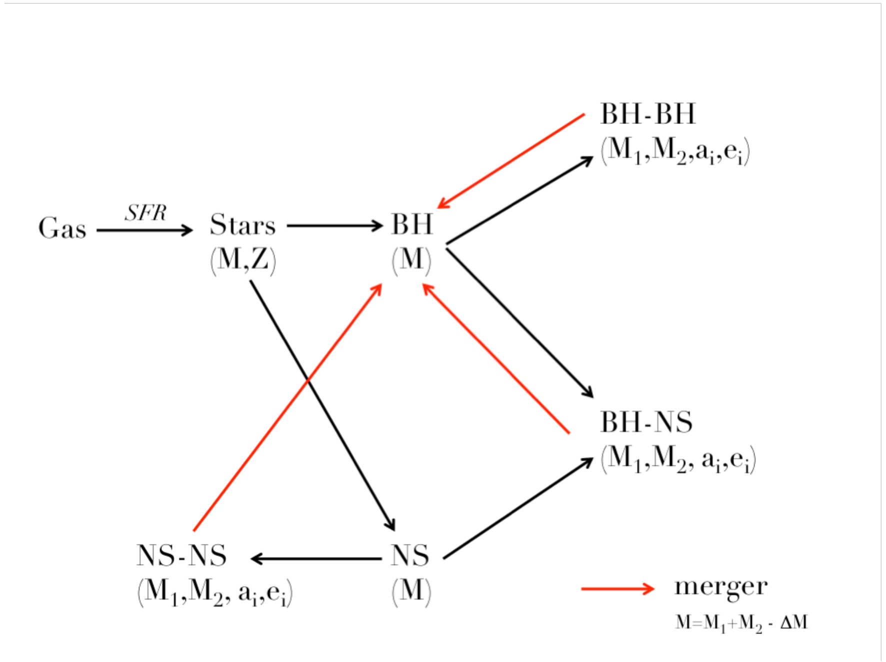
[Dvorkin, Vangioni, Silk, JPU, Olive, 2016]

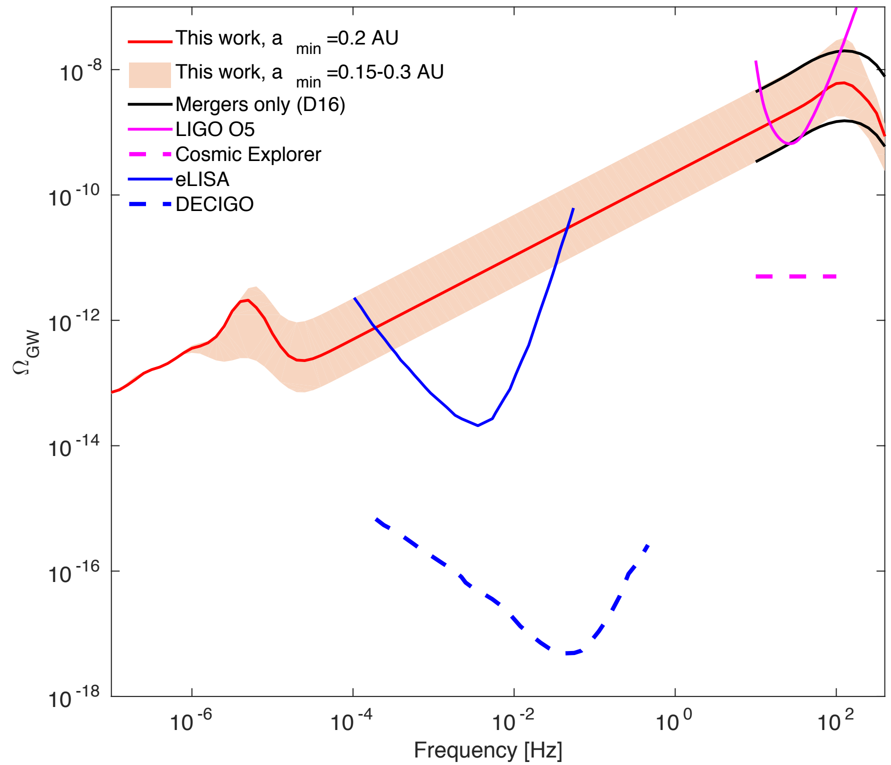
[see Talk by Elisabeth Vangioni]

Astrophysical GW background

Improvements:

- take into account the GW background arising from the inspiral phase (unresolved)
- describe the evolution of binary systems (NS & BH) as a system of interacting fluids





[Dvorkin, JPU, Vangioni, Silk, 2016]

Astrophysical GW background

Astrophysical observations include the detection of the radiation from

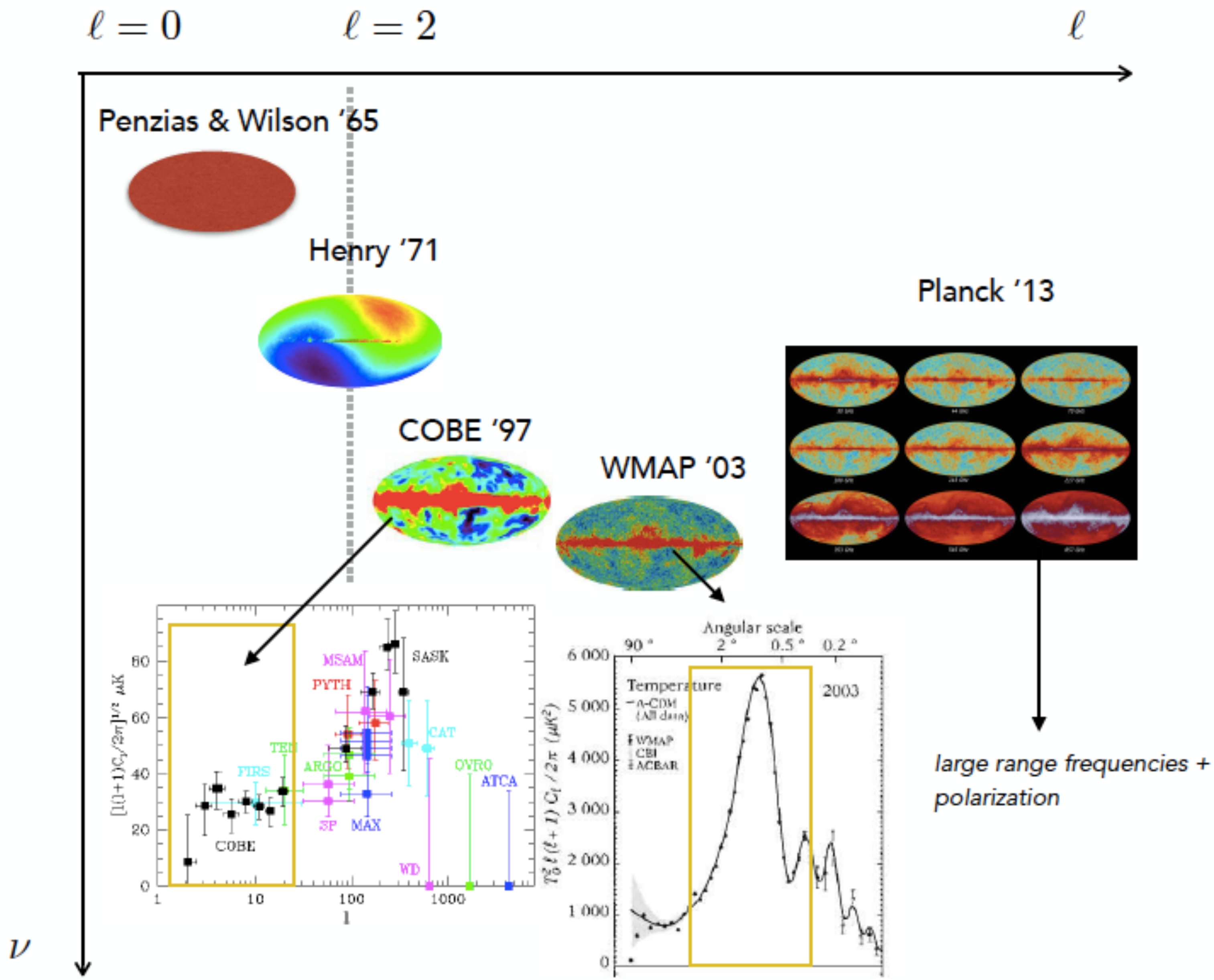
- resolved sources
- unresolved sources

EM radiation	CMB	IR extragalactic background
GW radiation	cosmological background	**astrophysical background**

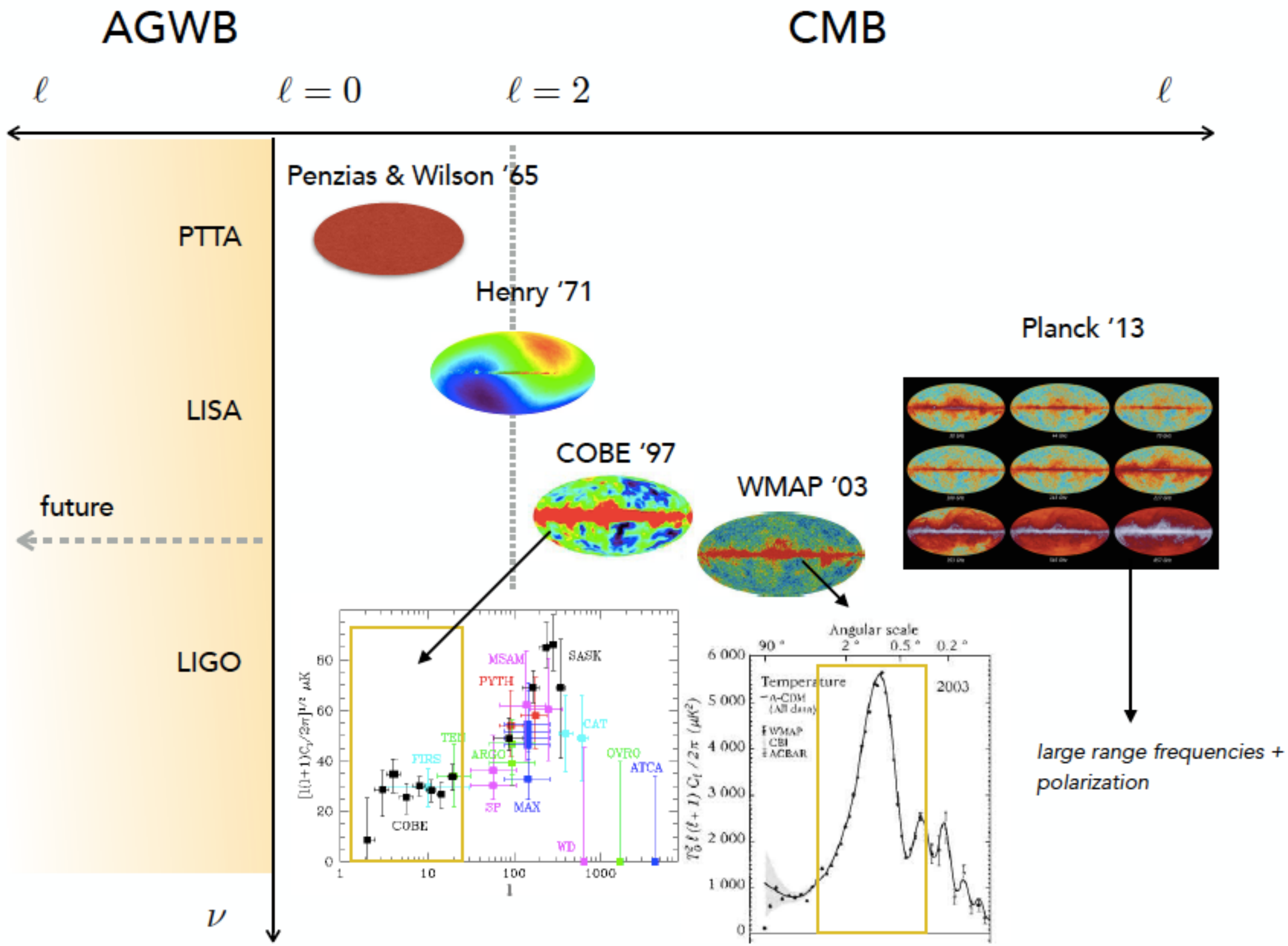
While the mean energy density of GW has been well studied, its anisotropies have not.

- far away from observation
- prospective

CMB



ℓ = 0
ℓ = 2
ℓ
ν
Penzias & Wilson '65
Henry '71
COBE '97
WMAP '03
Planck '13
SASK
MSAM
PYTH
CAT
TEN
FIRS
ARGO
OVRO
ATCA
SP
MAX
COBE
WD
Angular scale
90 °
2 °
0.5 °
0.2 °
Temperature
2003
WMAP
CBI
ACBAR
large range frequencies + polarization


AGWB
CMB
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Penzias & Wilson '65
PTTA
Henry '71
Planck '13
LISA
COBE '97
WMAP '03
future
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ν
large range frequencies +
polarization
MSAM
PYTH
SASK
TEN
FIRS
ARGO
CAT
OVRO
ATCA
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Angular scale
90°
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Temperature
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WMAP
CBI
ACBAR

AGWB

ℓ $\ell = 0$

PTTA

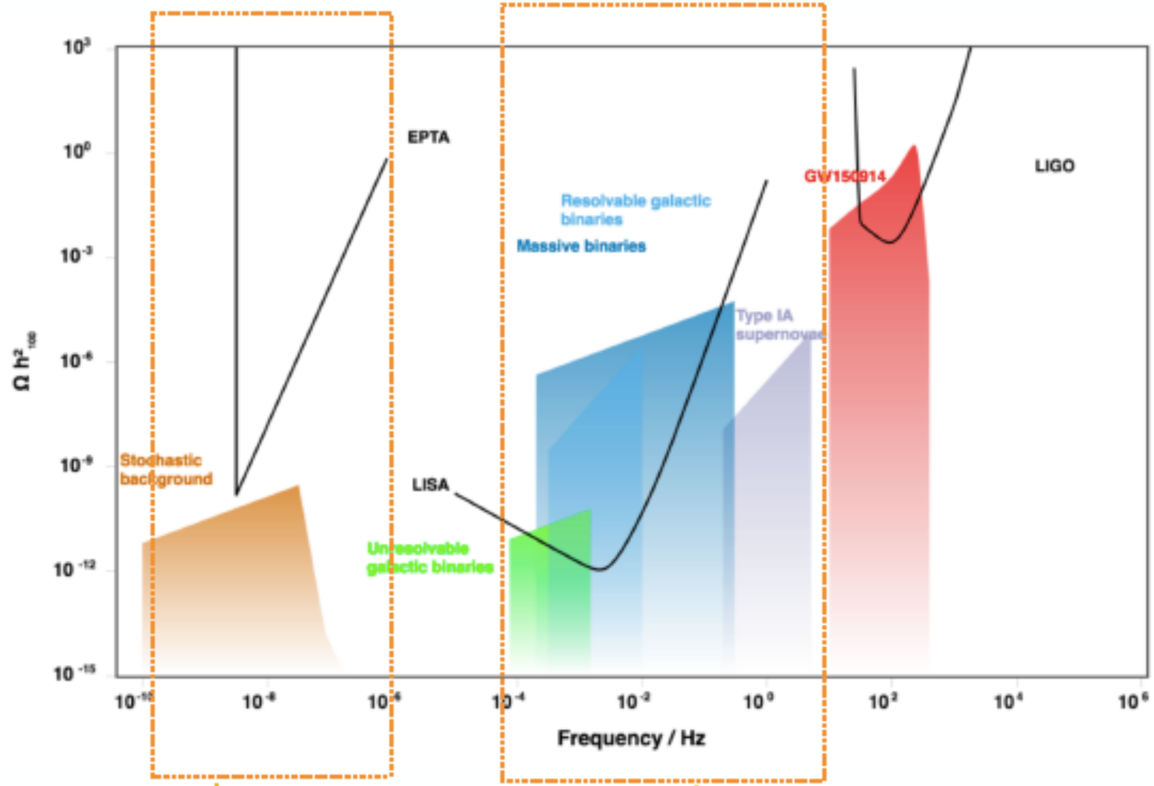
LISA

ET

future

LIGO

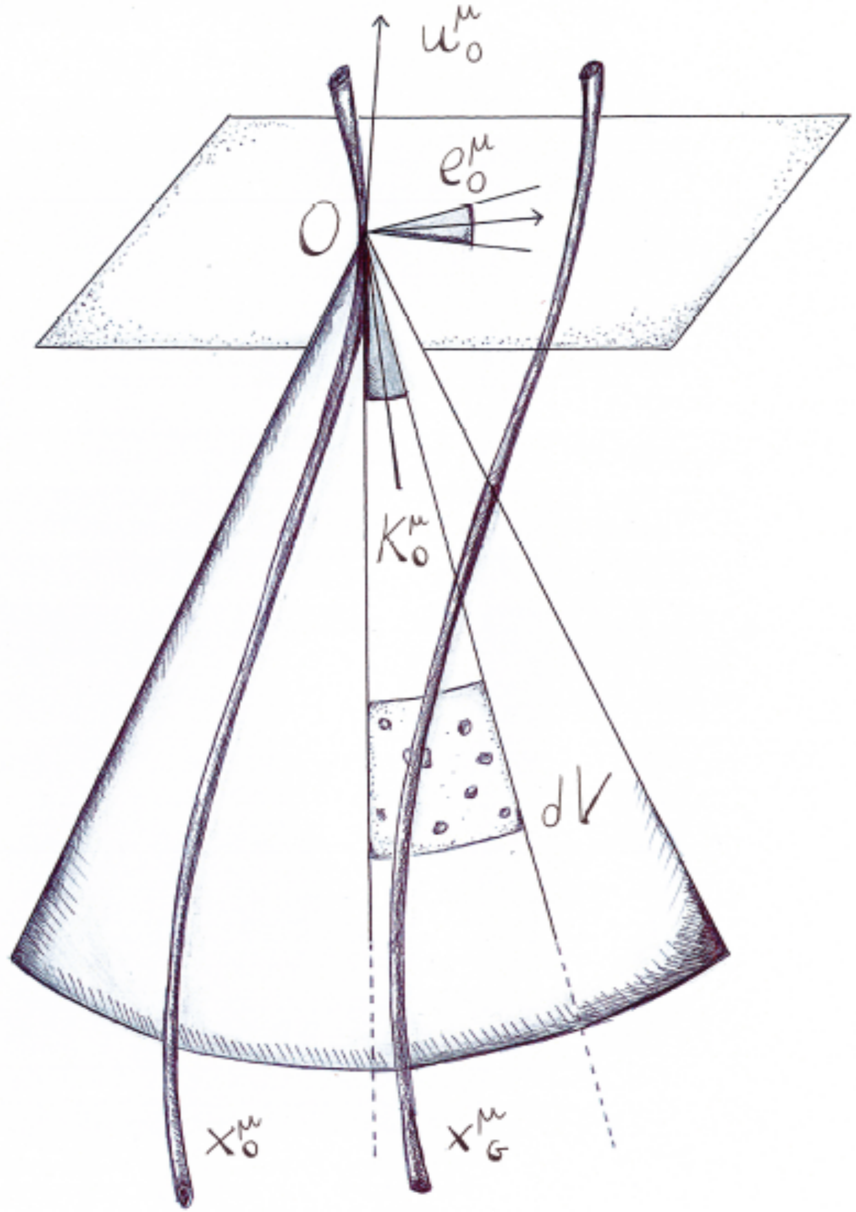
ν



cosmological background (inflation, cosmic strings…)

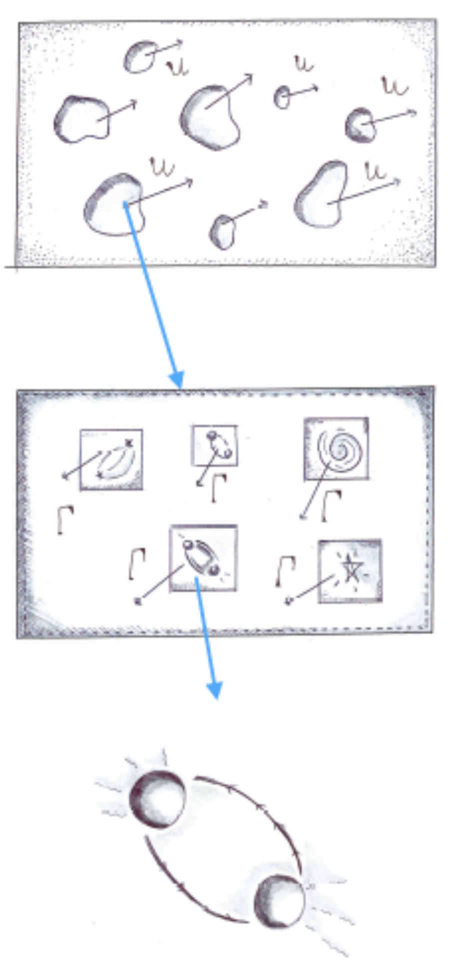
expected regime of frequencies and strain for **astrophysical stochastic background**

Anisotropies of the AGWB



Anisotropies of the AGWB

Three scales in our problem



(1) **cosmological scale**. The observer receives flux of GW in a solid angle around the direction of observation. Galaxies: point-like sources moving with the cosmic flow

cosmological approach

(2) **galactic scale**. A source -i inside a galaxy is characterized by parameters $\theta^{(i)}$ and is moving with velocity $\mathbf{\Gamma}$. Effective luminosity and frequency of a galaxy defined taking into account contributions sources

statistical approach

(3) **local scale**. Scale of single sources emitting GW inside a galaxy

astrophysical approach

Anisotropies of the AGWB: covariant result

Effective luminosity (per unit of effective frequency)

$$\mathcal{L}_G(\theta_G,\nu_G) = \mathcal{L}_G^I(\theta_G,\nu_G) + \boxed{\mathcal{L}_G^M(\theta_G,\nu_G) + \mathcal{L}_G^{SN}(\theta_G,\nu_G)}$$

$$\mathcal{L}_G^I(\theta_G,\nu_G) = \sum_{(i)}^{I} \int d\theta^{(i)} \mathcal{N}^{(i)}(\theta^{(i)},\theta_G) \int d^3\mathbf{\Gamma}\, f(\mathbf{\Gamma},\theta_G) \frac{dE_G^{(i)}}{dt_G d\nu_G}(\nu_G,\mathbf{\Gamma},\theta_G)$$

$$\boxed{\mathcal{L}_G^{M,SN}(\theta_G,\nu_G)} = \sum_{(i)}^{M,SN} \int d\theta^{(i)} \frac{d\mathcal{N}^{(i)}}{dt_G}(\theta^{(i)},\theta_G) \int d^3\mathbf{\Gamma}\, f(\mathbf{\Gamma},\theta_G) \frac{dE_G^{(i)}}{d\nu_G}(\nu_G,\mathbf{\Gamma},\theta_G)$$

$$\frac{d^2\rho_{\rm GW}}{d\nu_O d\Omega_O}(\nu_O,e_O) = \frac{1}{4\pi}\int d\lambda \int d\theta_G \boxed{\frac{1}{(1+z_G(\lambda))^3}\sqrt{p_\mu(\lambda)p^\mu(\lambda)}\, n_G(x^\mu(\lambda),\theta_G)}\,\boxed{\mathcal{L}_G(\nu_G,\theta_G)}$$

redshift

spatial displacement

effective luminosity of a galaxy

galaxy density

[Cusin, Pitrou, JPU, 2017]

Anisotropies of the AGWB: perturbed FL

$$\frac{d^2\rho_{GW}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O) = \frac{d^2\rho_{\overline{GW}}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \nu_O) + \boxed{\mathcal{E}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O)}$$

[cosmological contribution]

$$\frac{1}{4\pi}\int d\eta\, a^4 \int d\theta_G\, \bar{n}_G\, \mathcal{L}_G(\nu_G, \theta_G)\left[1 + b\delta_{CDM} + 4\Psi - 2\mathbf{e}\cdot\nabla v - 6\int_{\eta_O}^{\eta} d\eta'\,\dot{\Psi} + 2\partial_0 \bar{n}_G \int_{\eta_O}^{\eta} d\eta' \left((\eta-\eta')\dot{\Psi} - \Psi\right)\right]$$

[local physics contribution]

Doppler

integrated contributions

contribution local overdensity

	ρ_{GW}	κ	Δ
ρ_{GW}	$C_\ell(\nu_O)$	$B_\ell(\nu_O)$	$D_\ell(\nu_O)$
κ	$B_\ell(\nu_O)$	κ_ℓ	γ_ℓ
Δ	$D_\ell(\nu_O)$	γ_ℓ	Δ_ℓ

[Cusin, Pitrou, JPU, 2017]

Challenges and open questions

Theoretical:

- cosmic structure, specetime geometry, velocity fields
- GW production processes

Dependency in:

- Galaxy evolution & distribution
- binary formation rate & evolution
- properties of binary (PDF of eccentricity, semi-major axes,…)

Observational:

- not yet clear how it can be observed
- either provide information on local physics (mergers etc.) or new cosmological probe